

P425/2  
APPLIED MATHEMATICS  
3 HOURS

PRE- UNEB EXAMINATION 2024  
APPLIED MATHEMATICS  
3 HOURS

INSTRUCTIONS TO CANDIDATES

- Answer all questions in section A and any five questions in section B.
- In numerical work, take  $g$  to be  $9.8ms^{-2}$
- Any additional question(s) answered will not be marked.
- All necessary working must be shown clearly.
- Silent, non- programmable calculators and mathematical tables with a list of formula may be used.

## SECTION A

1. Events  $A$  and  $B$  are such that  $P(B) = \frac{1}{6}$ ,  $P(A \cap B) = \frac{1}{12}$ , and  $P(B/A) = \frac{1}{3}$ .

(a) Find

(i)  $P(A)$

(ii)  $P(A/B^c)$  (5 marks)

2. A particle moving with simple harmonic motion has speeds of  $5\text{ms}^{-1}$  and  $8\text{ms}^{-1}$  at distances 16m and 12 m respectively from its equilibrium position. Find the amplitude and the period of the motion. (5 marks)

3. A coin is biased such that when it is tossed, the head is twice as likely to occur as the tail. Find the probability that in seven tosses, there will be exactly two tails. (5 marks)

4. The table below shows the expenditure of a student in 2021 and 2022.

| Item          | Expenditure |        | Weight |
|---------------|-------------|--------|--------|
|               | 2021        | 2022   |        |
| Pocket money  | 40,000      | 50,000 | 3      |
| Text books    | 35,000      | 45,000 | 2      |
| Seminars      | 25,000      | 30,000 | 4      |
| Miscellaneous | 45,000      | 30,000 | 2      |

Taking 2021 as the base year,

(a) Calculate the weighted aggregate price index

(b) Calculate the average weighted price index (5 marks)

5. A river which is 50m wide flows with speed of  $3\text{ms}^{-1}$ . A boat must be set to cross a river to a point directly opposite the starting point. The speed of the boat relative to river is  $4\text{ms}^{-1}$ . Determine the;

(a) velocity with which the boat sets off to cross the river.

(b) time taken by the boat to cross the river. (5 marks)

6. The table below is an extract from tables of cotangents. Use linear interpolation/ extrapolation to find

| $\theta$        | $12'$  | $18'$  | $24'$  | $30'$  |
|-----------------|--------|--------|--------|--------|
| $\cot 28^\circ$ | 1.8650 | 1.8572 | 1.8495 | 1.8418 |

(a)  $\cot 28^\circ$

(b)  $\cot^{-1} 1.8533$  (5 marks)

7. An object of mass  $2000g$  is at rest on a plane inclined at  $25^\circ$  to the horizontal. The coefficient of friction between the object and the plane is  $\frac{2}{5}$ . What minimum force applied parallel to the plane would move the object up the plane? (5 marks)

8. A company has four production lines A, B, C and D producing large numbers of a certain item. Of the total daily production, 40% are produced by A, 20% by B, 15% by C and the rest by D. It is known that 2% of the items produced by A are defective. The corresponding proportions for B, C and D are 3%, 4% and 1% respectively. One item is chosen from the day's total output.

- (i) Find the probability that it is not defective.
  - (ii) Given that it is defective, find the probability that it was produced by A.
- (5 marks)

## SECTION B

9. The table below shows the temperature of patients recorded in a certain week.

|                   |       |       |       |       |       |       |
|-------------------|-------|-------|-------|-------|-------|-------|
| Temperature       | 10-15 | 15-20 | 20-30 | 30-50 | 50-55 | 55-60 |
| Frequency density | 1     | 2.4   | 3.0   | 2.0   | 2.2   | 0.2   |

- (a) Calculate the mode and standard deviation. (6 marks)
- (b) Draw a cumulative frequency curve and use it to determine the middle 60% of the temperature. (6 marks)

10. Given the equation  $x^3 - 9x + 3 = 0$ ,

- (a) Show that one of the positive roots of the equation lies between 2 and 3; Hence use linear interpolation to find the first approximation of the root. (4 marks)
- (b) Construct a flow chart based on the Newton- Raphson algorithm for computing the root of the equation in (a) above to 3 decimal places. Perform a dry run of your flow chart. (8 marks)

11. Forces of magnitude  $2N, 2N, 3N, 4N, 2\sqrt{2}N$  and  $\sqrt{2}N$  act along sides  $AB, BC, DC, AD, AC$  and  $DB$  respectively. Where the square is of side  $2m$ . Find the;

- (a) Resultant force. (8 marks)
- (b) Equation of the line of action of the resultant force and where it cuts the  $x$ -axis. (4 marks)

12. A machine cuts poles whose lengths are normally distributed with mean of  $4.2m$  and standard deviation of  $1.2m$ . If a random sample of 100 poles are selected, find the probability that a pole selected at random has its mean

sample length; (i) in the range 4.0m and 4.4m (4 marks)

(ii) more than 4.1m. (3marks)

(b) A survey of 150 households was asked how many people regularly eat bread for breakfast. The results of the survey are summarized below.

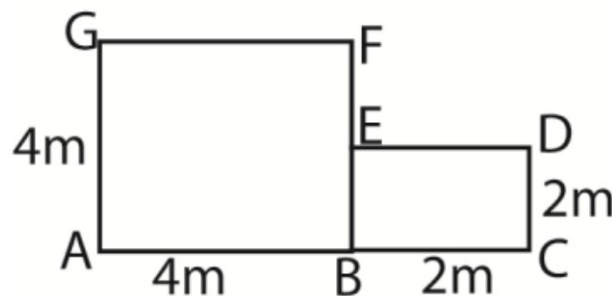
$$\sum x = 173, \sum x^2 = 355$$

Calculate the;

(i) unbiased estimate of the population variance (2 marks)

(ii) 97.5% confidence interval of the mean number of people who regularly eat bread for breakfast. (3 marks)

13. The diagram shows two uniform squares ABFG and BCDE joined together.



(a) Find the centre of gravity of the composite body. (8 marks)

(b) If the body is suspended freely at G, determine the angle AG makes with the horizontal. (4 marks)

14. The total number of radio taxi calls received at the control centre in a month is modelled by a random variable X (in tens of thousands of calls) having the probability density function,

$$f(x) = \begin{cases} cx, & 0 \leq x < 1 \\ c(2-x), & 1 \leq x < 2 \\ 0, & \text{otherwise} \end{cases}$$

(a) Sketch the function and hence show that the value of c is 1 (4 marks)

(b) Show that the cumulative distribution function of X is

$$F(x) = \begin{cases} 0; & x < 0 \\ \frac{1}{2}x^2; & 0 \leq x < 1 \\ 2x - \frac{1}{2}x^2 - 1; & 1 \leq x < 2 \\ 1; & x \geq 2 \end{cases}$$

(5 marks)

(c) Find the probability that the control centre receives between 8000 and 12000 calls in a month. (3 marks)

15. (a) Use the trapezium rule with six ordinates to estimate

$$\int_1^2 \left( \frac{2x}{7x^2 + 5} \right) dx$$

Give your answer correct to 3 significant figures. (5 marks)

(b) (i) Find the exact value of

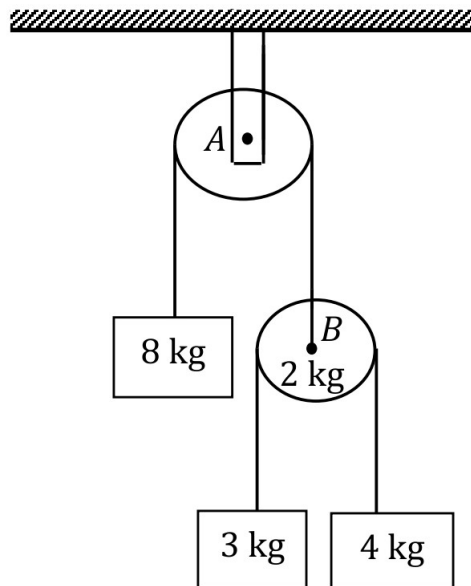
$$\int_1^2 \left( \frac{2x}{7x^2 + 5} \right) dx$$

Correct to 3 significant figures. (4 marks)

(ii) Calculate the percentage error in your calculation in (a) above. (2 marks)

(iii) Suggest how percentage error may be reduced. (1 mark)

16. The figure shows a light inextensible string passing over a smooth fixed pulley A, to one end of which is attached a mass of 8 kg and to the other end is attached pulley B of mass 2 kg. Over B passes a second light inextensible string which carries masses of 3 kg and 4 kg at its free ends.



The system is released from rest. Determine the:

(a) acceleration of the movable pulley, 3kg mass and 4kg mass. (8 marks)

(b) tensions in the strings. (4 marks)

P425/2  
APPLIED MATHEMATICS  
Paper 2  
November, 2024  
3 HOURS

PRE- UNEB EXAMINATIONS SET TWO 2024  
Uganda Advanced Certificate Of Education  
APPLIED MATHEMATICS  
Paper 2  
3 HOURS

INSTRUCTIONS TO CANDIDATES:

- Answer all the eight questions in section A and any five questions in section B.
- Any additional question(s) answered will not be marked.
- All necessary working must be shown clearly.
- Silent, non-programmable calculators and mathematical tables with a list of formula may be used.
- In numerical work, take  $g$  to be  $9.8ms^{-2}$

## SECTION A

1. A stone projected at an angle  $\alpha$  to the horizontal with speed,  $u \text{ ms}^{-1}$  just clears a vertical wall  $4\text{m}$  high and  $10\text{m}$  from the point of projection when traveling horizontally. Find the;

- (i) angle of projection  
(ii) time taken to reach that point. (5 marks)

2. Events A and B are such that  $P(A) = 0.4$ ,  $P(A/B) = 0.8$ , and  $P(A \cap B) = 0.25$ , find;

- (i)  $P(A \cap B^1)$ ,  
(ii)  $P(A^1 \cup B)$ ,  
(iii)  $P(B^1/A^1)$ . (5 marks)

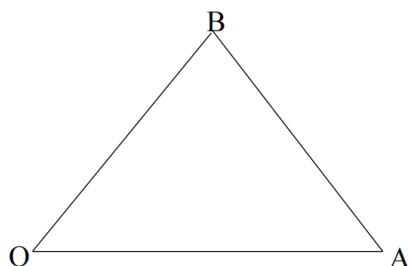
3. The temperature  $^{\circ}\text{C}$  of a room measured at an interval of 2 minutes were recorded as 55 and 52. If the initial temperature is 60. Use linear interpolation or extrapolation to find the;

- (a) Temperature after 5 minutes.  
(b) Time taken if the temperature is  $53.5^{\circ}\text{C}$ . (5 marks)

4. A boat travelling at a speed of  $18\text{kmh}^{-1}$  in the direction of  $N30^{\circ}E$  in still water is blown by wind moving at a speed of  $8\text{m/s}$  from the bearing of  $150^{\circ}$ . Calculate the speed and the course the boat will be steered. (5 marks)

5. Show graphically that the root of the equation,  $1 - e^{-x} = \sin x$  has a root between 2 and 3 and record it to 1 decimal place. (5 marks)

6. OAB is a uniform triangular lamina with vertices O(2,1), A(9,4) and B(6,8).



- (a) Find the coordinates of the centre of gravity of the lamina. (2 marks)  
(b) If the lamina is freely suspended at point B, calculate the angle the side BA makes with the horizontal. (3 marks)

7. The table below shows the trunk diameters in centimetres of 200 trees.

|                 |     |     |     |     |     |       |
|-----------------|-----|-----|-----|-----|-----|-------|
| Diameters (cm)  | 15- | 20- | 25- | 30- | 35- | 40-50 |
| Number of trees | 22  | 42  | 70  | 38  | 16  | 12    |

Determine;

- (a) the mean diameter.  
(b) the number of trees above 38cm of diameter. (5 marks)

8. A biased coin is thrice as likely to show heads as tails. If it is tossed 50 times, find the probability of obtaining;
- between 30 and 40 heads
  - above 40 heads.
- (5 marks)

### SECTION B (60 MARKS)

(Answer any five questions from this section. All questions carry equal marks.)

9. The marks for ten students at a certain school studying science were:

|           |    |    |    |    |    |    |    |    |    |    |
|-----------|----|----|----|----|----|----|----|----|----|----|
| Physics   | 35 | 24 | 37 | 41 | 39 | 30 | 32 | 27 | 31 | 41 |
| Chemistry | 30 | 17 | 35 | 39 | 38 | 24 | 27 | 24 | 29 | 40 |

- Plot a scatter graph and draw the line of best fit.
    - Use your graph to estimate the mark obtained in Physics if a student obtained 33 in Chemistry.

(7 marks)
  - Calculate the rank correlation coefficient for the above and comment at 1% level of significance.
- (5 marks)

10. (a) Show that  $x^2 - \sin x = 3$  has a root between 1.5 and 2. (4 marks)
- (b) Use linear interpolation to find the root of the equation in (a) above. Give your answer to 3 decimal places. (8 marks)

11. A force  $F = (t^2i + 3tj + 4k)\text{N}$  acts on a body of mass  $2\text{kg}$ . Initially the body is at rest at a point  $(2i - j + k)$  Find the:

- speed of the body after  $5\text{s}$ . (4 marks)
- distance of the body from the origin after  $2\text{s}$ . (5 marks)
- work done by the force after  $4\text{ seconds}$  (3 marks).

12. (a) The quantities  $a$  and  $b$  were measured with errors  $\delta a$  and  $\delta b$  respectively. Show that the percentage error in calculating  $z = a\sqrt{b}$  is  $(|\frac{\delta a}{a}| + \frac{1}{2}|\frac{\delta b}{b}|) \times 100\%$ . (7 marks)

- (b) Given that  $a = 2.5$  and  $b = 0.16$  were estimated with percentage errors of 4 and 5 respectively, calculate the absolute error in evaluating  $a\sqrt{b}$ . (5 marks)

13. The marks obtained by 2000 UNEB candidates in Maths paper 2 of a certain year were normally distributed with mean of 64. The records showed that 60% of the candidates scored above 50.

- Calculate the standard deviation of the candidates' marks (4 marks)
- Find the pass mark, if 75% of the candidates passed the paper. (4 marks)
- Calculate the number of candidates that scored between 45 and 55 marks (4 marks)

14. (a) A non-uniform ladder AB, 10m long and mass 8 kg, lies in limiting equilibrium with its lower end A resting on a rough horizontal ground and the upper end B resting against a smooth vertical wall. If the centre of gravity of the ladder is 3m from the foot of the ladder, and the ladder makes an angle of  $30^\circ$  with the horizontal, find the:

- (i) coefficient of friction between the ladder and the ground. (7 marks)
- (ii) reaction at the wall. (5 marks)

15. A container contains two boxes A and B. Box A contains 6 blue counters and 4 red counters while box B contains 4 blue counters and 3 red counters. A box is selected and two counters are removed from the box one at a time without replacement. If box A is thrice as likely to be picked as B;

- (a) Find the probability
  - (i) that the last counter chosen is red, given that box B was chosen.
  - (ii) of getting at most one blue counter. (6 marks)
- (b) Construct the probability distribution table for red counters and use it to find the;
  - (i) mean,
  - (ii)  $\text{var}(2X+5)$ . (6 marks)

16. At noon ship A is sailing due East at a constant velocity of  $20\text{kmh}^{-1}$ . At the same time ship B is sailing in the direction  $N60^\circ E$  at a constant velocity of  $15\text{kmh}^{-1}$ . At some instant, ship B is  $30\text{km}$  south of A. If they continue sailing with these velocities in these directions, Determine the;

- (a) time at shortest distance (5 marks)
- (b) shortest distance. (4 marks)
- (c) distance that ship A will have traveled in this time. (3 marks)

END